Exercise set 1 Algorithms and Complexity 2023 **Due 2-Oct-2023**

You may collaborate and submit answers in groups of at most two.
Good solutions are complete and concise.
Please e-mail to bouman@ese.eur.nl with subject "LNMB-AC Exercise Set 1".
Groups submissions should put the other group member in the submission e-mail's CC.

1. Harry Potter is looking for a Bowtruckle that has made itself invisble and is hiding in one of the vertices of a graph. Harry repeatedly points his magic wand at some vertex and casts the SEMI-REVILIO spell. When the spell hits the Bowtruckle for the first time, the Bowtruckle leaves its vertex and moves to an adjacent vertex; as the Bowtruckle moves silently and invisibly, Harry is not aware of the move. When the spell hits the Bowtruckle for the second time, it finally becomes visible to Harry.

   Harry wants too make the Bowtruckle visible while casting as few SEMI-REVILIO spells as possible to save battery for his wand. A vertex sequence is called *revealing*, if casting the SEMI-REVILIO spells at the vertices according to the sequence guarantees that the Bowtruckle eventually becomes visible (independently of its initial hiding place and move).

   Problem: REVEALING SEQUENCE
   Instance: A connected graph $G = (V, E)$; a bound k.
   Question: Does there exist a revealing vertex sequence of length at most k?

   And here are your tasks:

   (a) Prove that REVEALING SEQUENCE lies in NP.

   (b) Prove that REVEALING SEQUENCE is NP-hard.

2. An instance of the INDUCED MATCHING problem consists of an undirected graph $G' = (V', E')$ and an integer k'. The problem is to decide whether there exists a subset $U \subseteq V'$ with $|U| = 2k'$ such that the subgraph of G' induced by U consists of k' independent (e.g., non-overlapping) edges (and hence form an induced matching).

   Let us try to establish NP-hardness of INDUCED MATCHING by reducing the INDEPENDENT SET problem to it. We consider a graph $G = (V, E)$ and an integer k that form an instance of INDEPENDENT SET, and create the following instance of INDUCED MATCHING from it:

   - For every vertex $v \in V$, we create two vertices $a(v)$ and $b(v)$ in V'.
   - For every vertex $v \in V$, we create the edge $\{a(v), b(v)\}$ in E'.
   - For every edge $\{u, v\} \in E$, we create the two edges $\{a(v), b(u)\}$ and $\{a(u), b(v)\}$ in E'.
   - Finally we set $k' = k$'.

   What is left to prove is that the graph G contains an independent set of size k, if and only if the newly constructed graph G' contains an induced matching with k' edges.

   Your task is to decide whether the reduction outlined above is a correct polynomial time reduction: either complete the argument by proving the last claim, or provide a counter example