

Exercise set 3 Algorithms and Complexity 2023

Due 30 Oct 2023

You may collaborate and submit answers in groups of at most three. Good solutions are complete and concise. Please mail to T.vanderZanden@maastrichtuniversity.nl.

5. Consider the following branching algorithm for 3-SAT: we repeatedly pick a clause c with 3 unassigned variables v_i, v_j, v_k . We then branch on the following cases:
 - (a) We assign a truth value to v_i such that c is satisfied and leave v_j, v_k unassigned
 - (b) We assign the opposing truth value to v_i such that c is *not* satisfied and assign a truth value to v_j so that this assignment satisfies c and leave v_k unassigned
 - (c) We assign truth values to v_i, v_j so that c is *not* satisfied and then assign a truth value to v_k such that c is satisfied

Analyse the running time of this algorithm by writing down a recurrence relation and bound the running time as $O^*(c^n)$ for an as small as possible c .

6. Give an $O^*(k^{4k})$ -time algorithm that takes as input n points in the plane \mathbb{R}^2 and integer k , and determines whether there exist k straight lines such that every point is on some line.¹
7. A *triangle* of a graph $G = (V, E)$ is a triple $u, v, w \in V$ such that $\{u, v\}, \{v, w\}, \{u, w\} \in E$. A triangle partition is a partition of V into triangles, e.g., a set of triangles $T_1, \dots, T_{n/3}$ such that $T_i \cap T_j = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{n/3} T_i = V$. Give an algorithm that determines whether there is a triangle partition of a graph on n vertices in $O^*(2^n)$ time using polynomial space.
Note: If your algorithm uses exponential space, you still get 90% of the points.

¹Hint: (i) look at $k + 1$ points that are on one line to find a reduction rule. (ii) conclude something if n is too large when compared with k^2 and your reduction rule does not apply. (iii) design an $O^*(n^{2k})$ time algorithm.