

# Algorithms and Complexity (AC)

Extra Material: Introduction to Graphs

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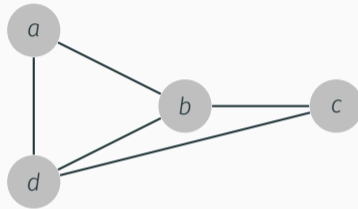
**Paul Bouman** & Tom van der Zanden (Based on slides by Gerhard Woeginger, Jesper Nederlof and Marie Schmidt)

September – November 2023

LNMB – Landelijk Netwerk Mathematische Besliskunde

## Basic concepts: Graphs

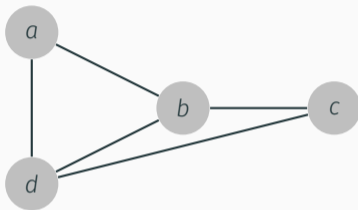
Graph: pair  $(V, E)$  where  $V$  is set of *vertices* and  $E$  is a set of pairs of vertices called *edges*



We often denote edges as  $\{u, v\}$  where  $u$  and  $v$  are the vertices that are connected by the edge. E.g.,  $\{a, b\}$ .

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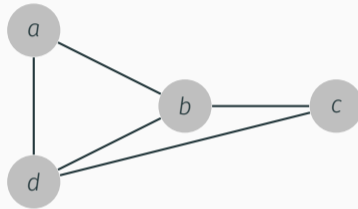


Also commonly used

- network  $\hat{=}$  graph
- node  $\hat{=}$  vertex
- arc  $\hat{=}$  (directed) edge

## Basic concepts: Graphs

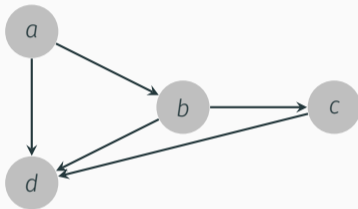
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In some applications, edges have *edge weights*.

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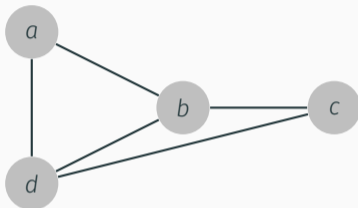


In some applications: edges are *directed*.

Then we write  $(a, b)$  to refer from the directed edge from  $a$  to  $b$ .

$(b, a)$  would be the edge in opposite direction.

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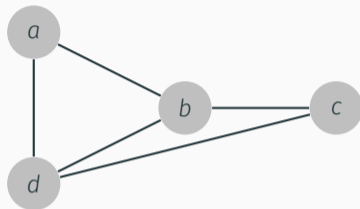


### Vocabulary:

- Two vertices are called adjacent, if they are connected by an edge.
- A vertex is called incident to an edge, if it is contained in the edge.

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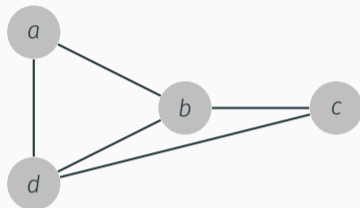
Graph encoding (undirected graphs)

Adjacency List

- $(a, b)$
- $(a, d)$
- $(b, c)$
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- $(c, d)$

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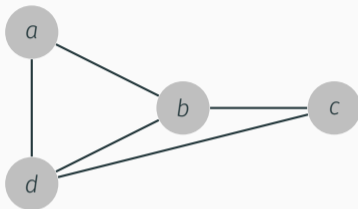
Adjacency Matrix

	$a$	$b$	$c$	$d$
$a$	0	1	0	1
$b$	0	0	1	1
$c$	0	0	0	1
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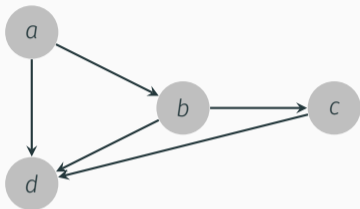
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Incidence Matrix

	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$
$a$	-1	-1	0	0	0
$b$	1	0	-1	-1	0
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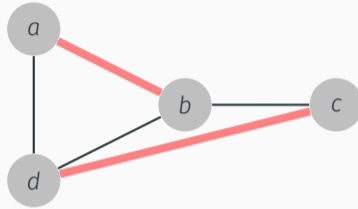
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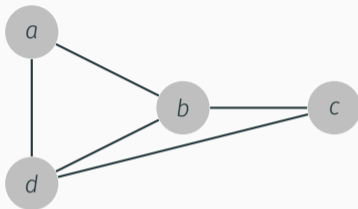


Some more vocabulary

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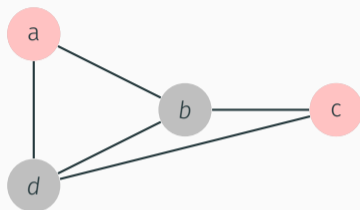


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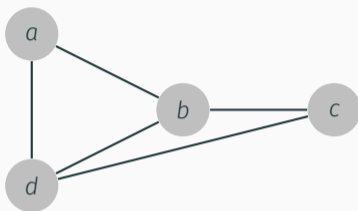


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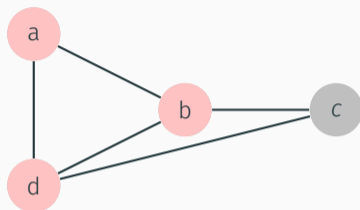


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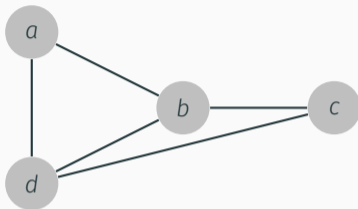
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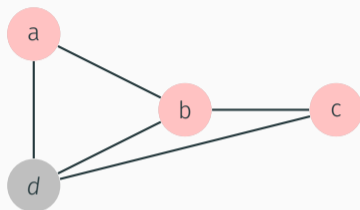
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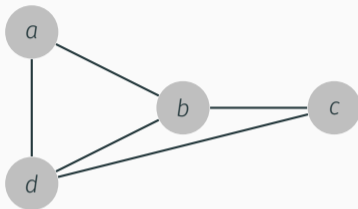
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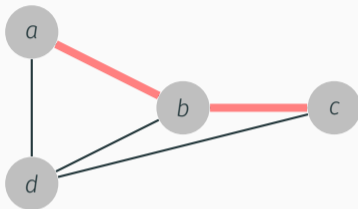
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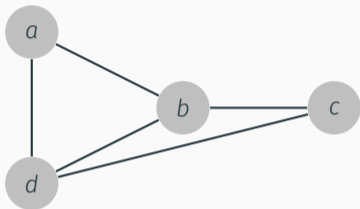
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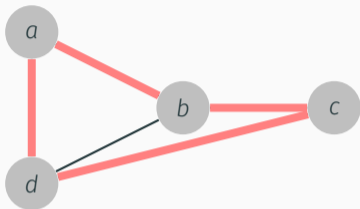
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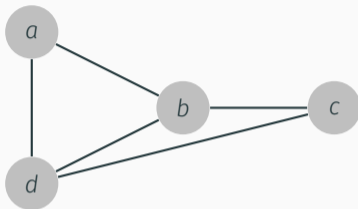
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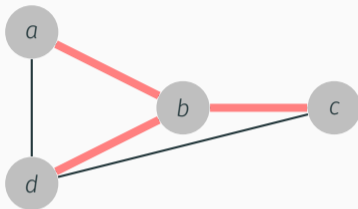
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**Tree:** connected edge-set without cycles