Algorithms and Complexity
Basic concept: Graphs
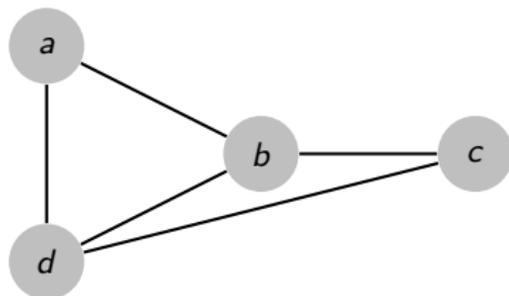
Marie Schmidt & Tom van der Zanden

(Based on slides by Gerhard Woeginger and Jesper Nederlof)
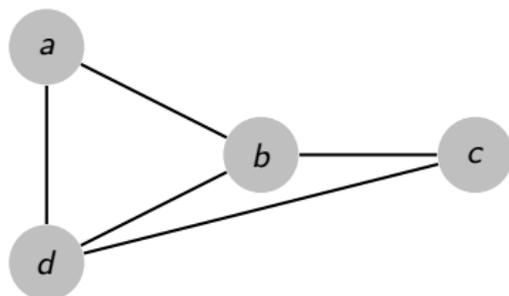
Landelijk Netwerk Mathematische Besliskunde

Graph: pair (V, E) where V is set of *vertices* and E is a set of pairs of vertices called *edges*

We often denote edges as $\{u, v\}$ where u and v are the vertices that are connected by the edge. E.g., $\{a, b\}$.
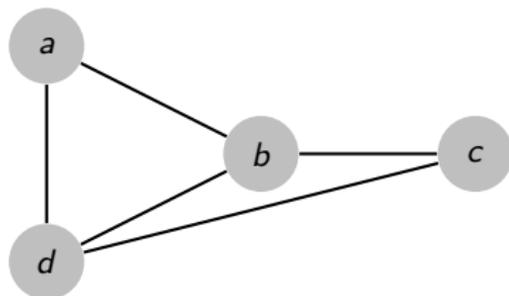
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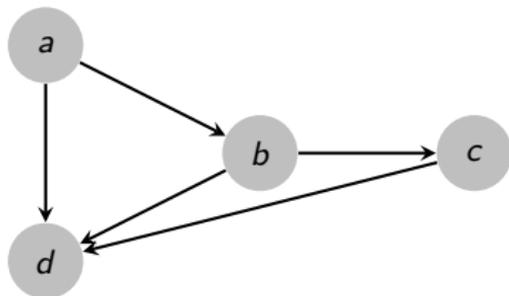

Also commonly used
- network $\triangleq$ graph
- node $\triangleq$ vertex
- arc $\triangleq$ (directed) edge

Graph: pair (V, E) where V is set of *vertices* and E is a set of pairs of vertices called *edges*

In some applications, edges have edge weights.

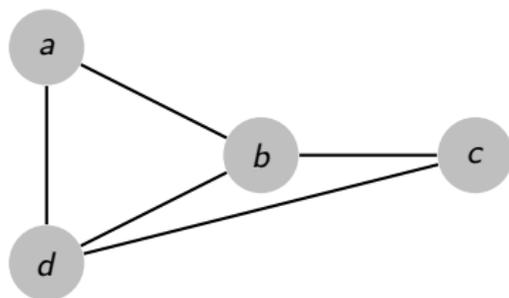Graph: pair (V, E) where V is set of *vertices* and E is a set of pairs of vertices called *edges*

In some applications: edges are directed.
Then we write (a, b) to refer from the directed edge from a to b.
(b, a) would be the edge in opposite direction.

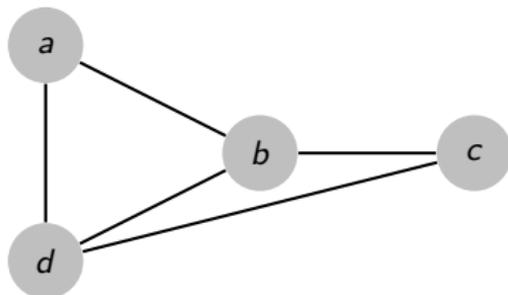Graph: pair (V, E) where V is set of *vertices* and E is a set of pairs of vertices called *edges*

Vocabulary:

- Two vertices are called adjacent, if they are connected by an edge.
- A vertex is called incident to an edge, if it is contained in the edge.

Basic concepts: graph
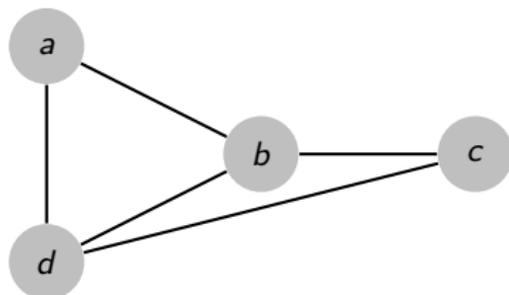
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Graph encoding (undirected graphs)

- Adjacency list: $(\{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\})$

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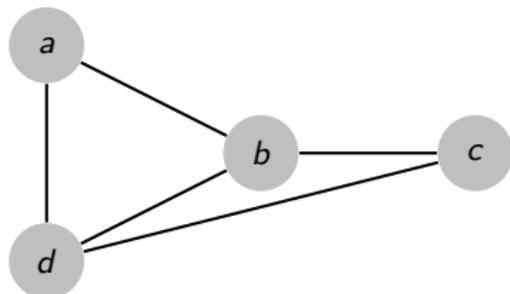**Graph encoding (undirected graphs)**

- Adjacency list: $(\{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\})$

- Adjacency matrix:

	a	b	c	d
a	0	1	0	1
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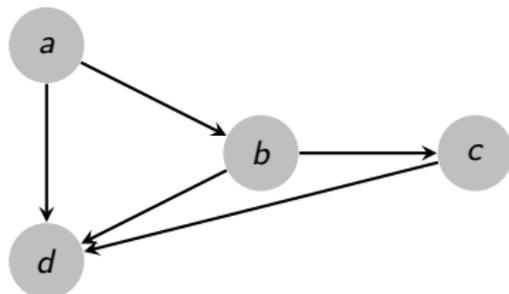- Adjacency list: $(\{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\})$

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- Incidence matrix:

	e_1	e_2	e_3	e_4	e_5
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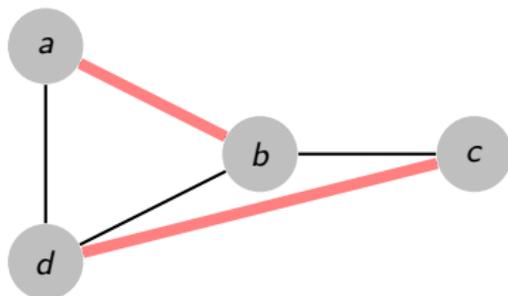- Adjacency list: $((a, b), (a, d), (b, c), (b, d), (c, d))$

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Some more vocabulary

Matching: set of non-adjacent edges (no two edges share a vertex), *perfect* if $|V|/2$ edges

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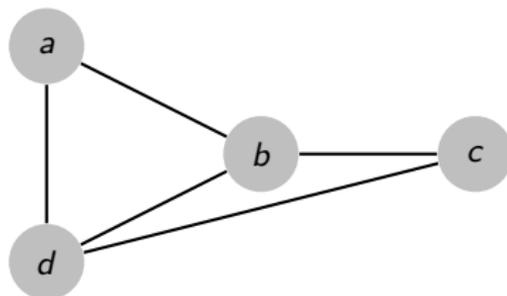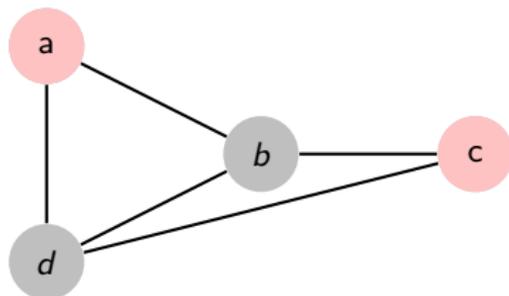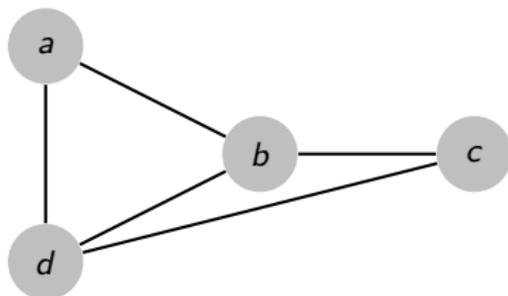Graph: pair (V, E) where V is set of *vertices* and E is a set of pairs of vertices called *edges*

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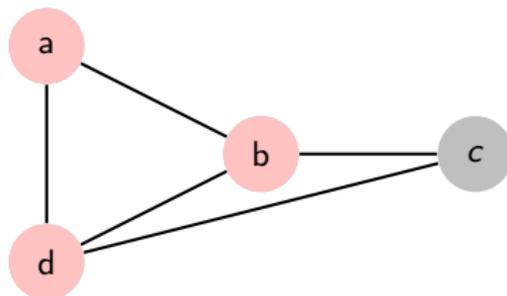

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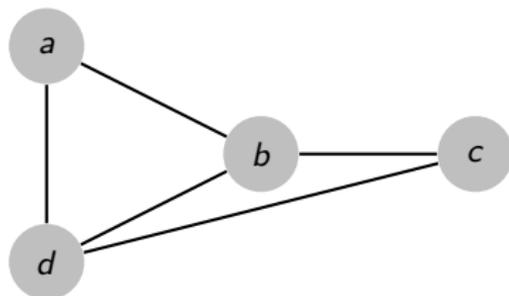

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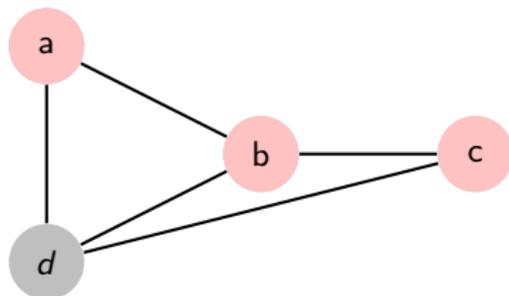Matching: set of non-adjacent edges (no two edges share a vertex), *perfect* if $|V|/2$ edges

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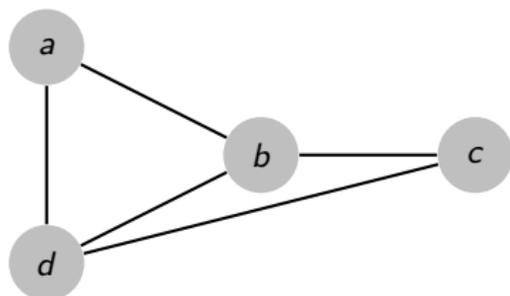Matching: set of non-adjacent edges (no two edges share a vertex), *perfect* if $|V|/2$ edges

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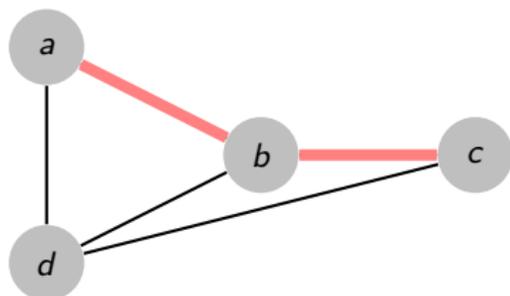Independent Set: set of pairwise non-adjacent vertices

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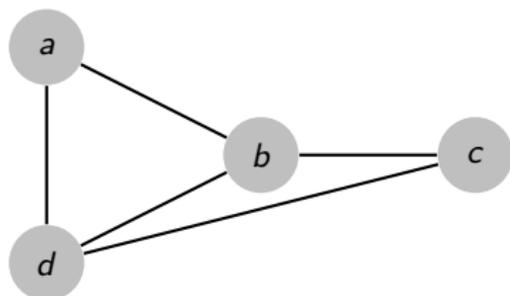Independent Set: set of pairwise non-adjacent vertices

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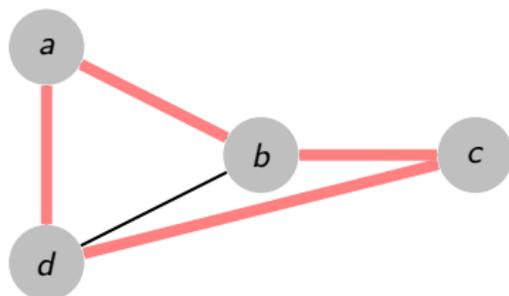
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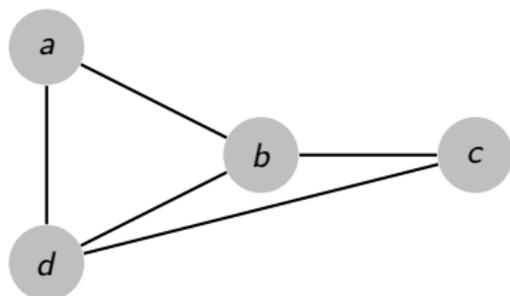
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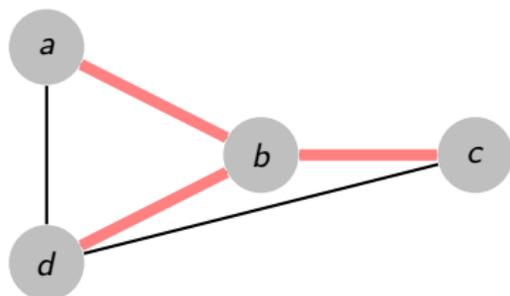
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